Grounding Logic on Defeasible, Material Implications

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For purposes of everyday reasoning and argumentation it seems fair to say that "the stove is switched on" implies "the stove is (or is getting) hot". This implication is defeated, however, by the addition of "there is a power outage". Examples like this suggest that (i) the implications we exploit in our everyday reasoning are typically defeasible, and (ii) sentences that don't contain any logical vocabulary can imply one another. Put another way: the implications that often matter for ordinary reasoning appear to be non-monotonic and material. In this paper, we shall not only take this appearance at face value, but shall furthermore suggest that we should understand the implications of classical logic as themselves *grounded* in these non-monotonic, material implications.

In order to show that classical logic may be understood as so grounded, we start with a non-monotonic consequence relation (with multiple conclusions), $\mid\!\sim_0$, ranging over sets of atomic sentences of a language $\mathfrak{L}_0$, i.e. $\mid\!\sim_0 \subseteq \mathcal{P}(\mathfrak{L}_0) \times \mathcal{P}(\mathfrak{L}_0)$. We then use familiar sequent rules to extend this pre-logical (and hence atomic) consequence relation to the language of propositional logic, $\mathfrak{L}$. The result is a non-monotonic but supra-classical relation $\mid\!\sim \subseteq \mathcal{P}(\mathfrak{L}) \times \mathcal{P}(\mathfrak{L})$ that is a conservative extension of the pre-logical consequence relation. Thus, we arrive at classical logic (as a fragment) simply by introducing standard logical connectives into our atomic language.

Our project may be best understood by examining three criteria of adequacy that guide the construction of our logic:

1. Everyday, pre-logical implications should be treated as most basic. While virtually all extant non-monotonic logics try to understand defeasible reasoning in terms of some familiar logic plus some additional structure (e.g. default rules, etc.), we reverse the order of explanation and understand familiar logics as grounded in non-monotonic, material implications. We therefore start with a non-monotonic, pre-logical consequence relation and extend it via the introduction of logical connectives. This can be done without resorting to (among other things) negative (or anti-)sequents, the addition of default rules to the object language, hypersequents, labeled sequents, or any of the other devices most proof-theoretic formulations of non-monotonic logics use. Our logic is therefore much more parsimonious than, e.g., many formulations of default logic.[1]

2. The behavior of logical connectives must arise *naturally* out of the pre-logical consequence relation. The simple rules we use to extend our pre-logical, material consequence relation should *on their own* produce the structural features we hope for. Krause, Lehman and Magidor (KLM),[2] for example, have famously formulated and advocated for five properties that all non-monotonic logics should have: reflexivity, cut, cautious monotonicity, left logical equivalence, and right weakening. In our sysem reflexivity arises naturally from the stipulation that our pre-logical consequence relation satisfy reflexivity in arbitrary atomic contexts. Reflexivity in the pre-logical (i.e. atomic) consequence relation is therefore preserved in the extended logically complex consequence relation by our sequent rules. Left logical equivalence also arises naturally. Our system, however, does not satisfy cut,[3] cautious monotonicity, nor right weakening. We explain why each of these ought to be rejected.

The general strategy of attack is that, firstly, none of these three properties are preserved by plausible connective rules (i.e. the extended consequence relation need not have them even if the pre-logical consequence relation does) and, secondly, that these properties are incompatible with very natural and desirable features of the logical connectives. Cut, for example, makes it impossible to have a conditional that obeys a deduction theorem (in the context of accepting reflexivity and rejecting monotonicity).[4] We, therefore, show along the way why KLM's argument for rejecting the deduction theorem in non-monotonic settings is not convincing. Moreover, we show that right weakening (which many take to be a correlate of left-logical

[1] See for example: Milnikel, Robert Saxon (2005). "Sequent calculi for skeptical reasoning in predicate default logic and other nonmonotonic logics", *Annals of Mathematics and Artificial Intelligence* 44, 1-2: pp. 1-34; Bonatti, Piero A and Olivetti, Nicola (1997). "A sequent calculus for skeptical default logic", in *International Conference on Automated Reasoning with Analytic Tableaux and Related Methods*, pp. 107–121; Reiter, R. (1980). "A logic for default reasoning," *Artificial Intelligence* 13, 1–2: pp. 81–132.

[2] Krause, S., Lehmann, D., and Magidor, M. (1990). "Nonmonotonic reasoning, preferential models and cumulative logics", *Artificial Intelligence* 44: pp. 167–207.

[3] Though one version of our system can be easily modified such that cut also *naturally* arises out of the pre-logical consequence relation. Nevertheless, David Ripley has provided independent and compelling reasons for rejecting cut. See Ripley, David (2015). "Anything goes." *Topoi* 34.1: pp. 25-36. We are also able to unproblematically add a transparent truth predicate to our system. See Ripley, David (2013). "Paradoxes and failures of cut." *Australasian Journal of Philosophy* 91.1: pp. 139-164.

[4] By deduction theorem we mean: $\Gamma, A \mid\!\sim B \Leftrightarrow \Gamma \mid\!\sim A \to B$.

equivalence), belies basic facts about how negation ought to behave. Namely, the fact that $\Gamma \mid\!\sim \neg A$ should hold just in case $\Gamma, A \mid\!\sim \emptyset$ (where the empty set on the right expresses that the set on the left is incoherent). That is, negation is used to express that a set of sentences are incompatible. Right weakening undermines this behavior of negation—given natural rules for conjunction, left logical equivalence, and non-monotonicity of incoherence. Cautious monotonicity leads to implausible behavior of disjunctions on the left. In particular, it renders what KLM call (Or)[5] irreversible.

Since we work in a Gentzen system, rejecting right weakening amounts to making the logic paraconsistent. We think of this as an advantage; especially because we can recover explosion in the classical fragment of our logic.

3. Features which arise naturally in some but not all fragments of the consequence relation should be markable in the object language. That is: we should be able to introduce expressions into the object language that keep track of these features. One interesting structural feature, for example, that arises from our sequent rules is that our system is supra-classical as it stands. That is, classical logic is a proper part of our non-monotonic consequence relation. This part, as a result, obviously behaves monotonically. Because we are arguing that logic should be grounded on pre-logical, non-monotonic consequence relations, we want to be able to pick out the monotonic and the classical fragments of our consequence relation in the object language. We do this by introducing a modal operator that marks consequences which hold monotonically, i.e. cannot be defeated. And we develop a similar operator that marks classical consequences (which are a proper part of the monotonic ones).

The result of all this is a sequent calculus that extends pre-logical, non-monotonic consequence relations to a language with logical vocabulary. The sequent rules are familiar and straightforward. The extended consequence relation is paraconsistent, non-monotonic, non-transitive, but supra-classical. We can, furthermore, keep track of monotonicity and classicality using our newly introduced operators. We can thus understand classical logic as simply a part of what happens when you introduce standard logical connectives into an atomic language with a non-monotonic consequence relation.

[5]See p. 190 of KLM:

$$\frac{\alpha \mid\!\sim \gamma \qquad \beta \mid\!\sim \gamma}{\alpha \vee \beta \mid\!\sim \gamma} \text{(Or)}$$